ANALYTICAL INVESTIGATION AND OPERATIONAL RULE FOR DOUBLE LAPLACE TRANSFORM

DISSERTATION SUBMITTED IN
PARTIAL FULLFILMENT OF REQUIREMENT FOR THE AWARD
OF THE DEGREE OF

MASTER OF SCIENCE
IN
MATHEMATICS & COMPUTING

BY
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&
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INDIAN INSTITUTE OF TECHNOLOGY(ISM) DHANBAD
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MAY, 2019

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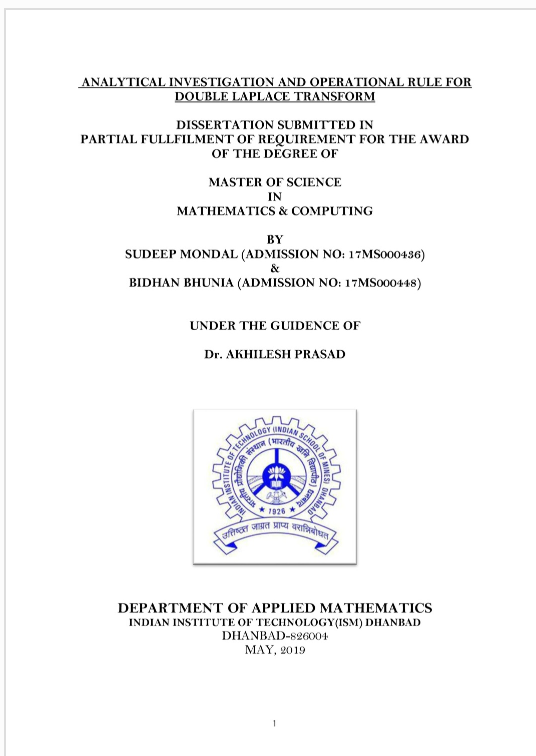

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CHAPTER-1

DOUBLE LAPLACE TRANSFORM

1.1 Introduction:

We are familiar with the Laplace Transform of function one variable, its properties application. But there is almost nothing about the double Laplace Transform, its properties & applications.
Double Laplace Transform is the most powerful methodology to solve partial differential equations. It is very useful for solving Telegraphic, Integro-Differential & Wave Equations in many branches of Applied Mathematics as well as Engineering.

1.2 Definition:

The double Laplace transform has been defined by Estrin and Higgins in 1951 [1] as:

$$\overline{\overline{g}}(r,s) = L_t L_x\left[g(x,t)\right] = \int_0^\infty e^{-st}\int_0^\infty e^{-px} g(x,t)\, dxdt = L_x L_t\left[g(x,t)\right] \quad ...(1),\ r,s \in \mathbb{C},$$

Where $g(x,t)$ is the bivariate function of $(x,t)\in[0,\infty)\times[0,\infty)$. Provided the improper integral converges.

Definition of Inverse Double Laplace Transform [1]:

Inverse double Laplace transform with respect to 'r' of $g(x,t)$ is given by

$$L_x^{-1}[\overline{\overline{g}}(r,s)] = \frac{1}{2\pi i}\int_{Br.} e^{xp}\,\overline{\overline{g}}(r,s)dr \qquad (1.1)$$

This inversion gives rise the single transformed function $\overline{g}(x,s)$. Then second inversion w.r.t "s" gives the required function $g(x,t)$

$$L_t^{-1}[\overline{g}(x,s)] = \frac{1}{2\pi i}\int_{Br.} e^{st}\,\overline{g}(x,s)ds \qquad (1.2)$$

Combining both of the successive inversions (2) and (3) yields,

$$L_t^{-1}L_x^{-1}[\overline{\overline{g}}(r,s)] = \frac{1}{2\pi i}\int_{Br.} e^{ts}\frac{1}{2\pi i}\int_{Br.} e^{xp}\,\overline{\overline{g}}(r,s)\ dpds = L_x^{-1}L_t^{-1}[\overline{\overline{g}}(r,s)]$$

1.3 Existence Condition For Double Laplace Transform [2]:

Let us assume that a function $g(x,t)$ is defined on $[0,\infty)\times[0,\infty)$ which is continuous and as well as in exponential order.

i.e., for some a, bϵ R.

Consider $\sup\limits_{x>0,\ t>0}\left[\frac{|g(x,t)|}{e^{ax+bt}}\right] < \infty$.

In this case, the double Laplace transform of $g(x,t)$

$$L_t L_x\{g(x,t)\} = \overline{\overline{g}}(r,s) = \int_0^\infty e^{-st} \int_0^\infty e^{-rx} g(x,t)\,dx\,dt$$

Holds $\forall$ r $> a$ & $s > b$ and is infinitely differentiable w.r.t $r > a$ & $s > b$.

Study of these functions are supposed to be of exponential order.

Examples:

(1) Suppose h(x,t)=1 for x>0, t>0, then

$$L_t[h(x,t)] = \frac{1}{s} = \overline{h}(x,s)$$

$$L_x[\overline{h}(x,s)] = \frac{1}{rs} = \overline{\overline{h}}(r,s)$$

$$\Rightarrow L_x L_t[1] = \frac{1}{rs}.$$

(2) Suppose $h(x,t) = e^{(ax+bt)}$ for all x, t>0, then

$$L_t[h(x,t)] = \overline{h}(x,s) = e^{ax}\frac{1}{s-b}$$

$$L_x[\overline{h}(x,s)] = \overline{\overline{h}}(r,s) = \frac{1}{(r-a)(s-b)}$$

$$\Rightarrow L_x L_t[e^{ax+bt}] = \frac{1}{(r-a)(s-b)}.$$

(3) Suppose $h(x,t) = e^{i(ax+bt)}$ for all x,t>0 , then

$$L_t[h(x,t)] = \overline{h}(x,s) = e^{iax}\frac{1}{s-ib}$$

$$L_x[\overline{h}(x,s)] = \overline{\overline{h}}(r,s)\frac{1}{(r-ia)(s-ib)} = \frac{(rs-ab)+i\,(as+br)}{(r^2+a^2)(s^2+b^2)}$$

$$\Rightarrow L_x L_t[e^{i(ax+bt)}] = \overline{\overline{h}}(r,s) = \frac{(rs-ab)+i(as+br)}{(r^2+a^2)(s^2+b^2)}.$$

So,

$$L_x L_t[\cos(ax+bt)] = \frac{(rs-ab)}{(r^2+a^2)(s^2+b^2)}$$

$$L_x L_t[\sin(ax+bt)] = \frac{(as+br)}{(r^2+a^2)(s^2+b^2)}.$$

Some Suitable Double Laplace Transforms are given in tabular form

$g(x, t)$	$L_x L_t[g(x, t)] = \bar{\bar{g}}(p,s)$
1	$\frac{1}{ps}$
$e^{(ax+bt)}$	$\frac{1}{(p-a)(s-b)}$
$\cos(ax+bt)$	$\frac{(ps-ab)}{(p^2+a^2)(s^2+b^2)}$
$\sin(ax+bt)$	$\frac{(as+bp)}{(p^2+a^2)(s^2+b^2)}$
$\cosh(ax+bt)$	$\frac{1}{2}\left[\left(\frac{1}{p-a}\right)\left(\frac{1}{s-b}\right)+\left(\frac{1}{p+a}\right)\left(\frac{1}{s+b}\right)\right]$
$\sinh(ax+bt)$	$\frac{1}{2}\left[\left(\frac{1}{p-a}\right)\left(\frac{1}{s-b}\right)-\left(\frac{1}{p+a}\right)\left(\frac{1}{s+b}\right)\right]$
$e^{-ax-bt} f(x,t)$	$\bar{\bar{f}}(p+a, s+b)$
$x^m t^n$	(i) $\frac{m!n!}{p^{m+1}s^{n+1}}$;where m, n $\in \mathbb{Z}^+$ (ii) $\frac{\Gamma(m+1)}{p^{m+1}}.\frac{\Gamma(n+1)}{s^{n+1}}$; where m>1, n>-1
$\frac{1}{\sqrt{xt}}$	$\frac{\pi}{\sqrt{ps}}$
$J_0(a\sqrt{xt})$	$\frac{4}{(4ps+a^2)}$
(i) $erf\left(\frac{x}{2\sqrt{t}}\right)$ (ii) $erf\left(\frac{t}{2\sqrt{x}}\right)$	(i) $\left(\frac{1}{p\sqrt{s}}\right)\frac{1}{\left(p+\sqrt{s}\right)}$ (ii) $\left(\frac{1}{s\sqrt{p}}\right)\frac{1}{\left(s+\sqrt{p}\right)}$

1.4 Basic Properties of Double Laplace Transforms [3]

(1) $L_x L_t[e^{-ax-bt} f(x,t)] = \overline{\overline{f}}(r+a, s+b)$

(2) $L_x L_t[f(ax)g(bt)] = \frac{1}{ab}\overline{\overline{f}}\left(\frac{r}{a}\right)\overline{\overline{g}}\left(\frac{s}{b}\right), a>0, b>0$

(3) $L_x L_t[f(x)] = \frac{1}{s}\overline{\overline{f}}(r)$; $L_x L_t[g(t)] = \frac{1}{p}\overline{\overline{g}}(s)$

(4) $L_x L_t[f(x+t)] = \frac{1}{r-s}[\overline{\overline{f}}(r) - \overline{\overline{f}}(s)]$

(5) $L_x L_t[f(x-t)] = \frac{1}{r+s}[\overline{\overline{f}}(r) + \overline{\overline{f}}(s)]$, when f is an even function

$= \frac{1}{r+s}[\overline{\overline{f}}(r) - \overline{\overline{f}}(s)]$, when f is a odd function

(6) $L_x L_t[f(x)H(x-t)] = \frac{1}{s}[\overline{\overline{f}}(r) - \overline{\overline{f}}(r+s)]$

(7) $L_x L_t[f(x)H(t-x)] = \frac{1}{s}[\overline{\overline{f}}(r+s)]$

(8) $L_x L_t[f(x)H(x+t)] = \frac{1}{s}[\overline{\overline{f}}(r)]$

(9) $L_x L_t[H(x-t)] = \frac{1}{r(r+s)}$

(10) $L_x L_t\left[\frac{\partial u}{\partial x}\right] = p\overline{\overline{u}}(r,s) - \overline{u}(0,s)$

(11) $L_x L_t\left[\frac{\partial u}{\partial t}\right] = s\overline{\overline{u}}(r,s) - \overline{u}(p,0)$

(12) $L_x L_t\left[\frac{\partial^2 u}{\partial x^2}\right] = r^2\overline{\overline{u}}(r,s) - r\overline{u}(0,s) - \overline{u}_x(0,s)$

(13) $L_x L_t\left[\frac{\partial^2 u}{\partial t^2}\right] = s^2\overline{\overline{u}}(r,s) - s\overline{u}(r,0) - \overline{u}_t(r,0)$

(14) $L_x L_t\left[\frac{\partial^2 u}{\partial x \partial t}\right] = rs\overline{\overline{u}}(p,s) - s\overline{u}(0,s) - r\overline{u}(r,0) + u(0,0)$.

1.5 Some important results: [4 & 9]

Statement (1):

Suppose $\varphi(x,t)$ is a bivariate continuous function in the positive region of $R=\{(a,b):0<x<\infty,0<t<\infty\}$. If the integral

$$\int_0^\infty\int_0^\infty e^{-px-st}\varphi(x,t)dxdt$$

Converges at $p=p_0, s=s_0$ then integral converges for $p>p_0, s>s_0$.

Statement (2):

If $L_x L_t[g(x,t)]=\overline{\overline{g}}(p,s)$ and

$$f(x,t)=\int_0^x\int_0^t g(u,v)dvdu \quad , \tag{1.3}$$

then

$$L_x L_t\{\int_0^x\int_0^t g(u,v)dvdu\}=\frac{\overline{\overline{g}}(p,s)}{ps} \quad . \tag{1.4}$$

Proof:

Denote $h(x,t)=\int_0^t g(x,v)dv$. By fundamental theorem of calculus

$$h_t(x,t)=g(x,t) \tag{1.5}$$

And

$$h(x,0)=0. \tag{1.6}$$

Invoking double Laplace transform in (1.5), we obtain

$$s\overline{\overline{h}}(p,s)-\overline{h}(p,0)=\overline{\overline{g}}(p,s) \tag{1.7}$$

And Single Laplace Transform of equation (1.6)

$$\overline{h}(p,0)=0$$

Then equation (1.7) becomes,

$$\bar{\bar{h}}(p,s) = \frac{\bar{\bar{g}}(p,s)}{s} \quad . \tag{1.8}$$

From (1.3)

$$f(x,t) = \int_0^x h(u,t)du$$

$f_x(x,t) = h(x,t)$ And $f(0,t) = 0,$

$$p\bar{\bar{f}}(p,s) - \bar{f}(0,s) = \bar{\bar{h}}(p,s)$$

Now using (1.8) and (1.3), we get

$$L_x L_t\{g(u,v)dvdu\} = \frac{\bar{\bar{g}}(p,s)}{ps} \quad .$$

CHAPTER 2

THE TELEGRAPHIC EQUATION

2.1 The Telegraph Equation [5]:

Model of extremely small piece telegraph wire is taking as an electrical circuit, which obey the resistor of resistance Rdx and an inductance of coil is Ldx. Using the current flow through the wire of amount $i(x,t)$ and voltage through resister is $iRdx$.When that passing in-between the coil is $i_t Ldx$. At any instance "t" with the position "x" is denoted by $h(x,t)$. So the change of voltage between two ends of a piece of wire is

$$dh = (iR - itL)dx$$

Let us suppose that current $i(x,t)$ can be emitted from the wire to ground, either by resisters with conductance Gdx or by a capacitor with capacitance Cdx. The quantity that emitted from the resistor is $hGdx$. As the charge on the capacitor is $q = hCdx$, the quantity that emitted through the capacitor is $u_t Cdx$.

Finally,

$$di = -hGdx - h_t Cdx.$$

On dividing both sides in the last expression by dx and assuming the limit $dx \to 0$, the required differential equations are as follows

$$h_x = Ri + Li_t = 0 \text{...........}(k1)$$
$$Ch_t + Gh + i_x = 0 \text{...........}(k2)$$

Solving $\frac{\partial}{\partial t}(k2)$ for

$$i_{xt} = -Ch_{tt} - Gh_t$$

And substituting the result into $\frac{\partial}{\partial t}(k1)$ gives

$$h_{xx} + Ri_x + L(-Ch_{tt} - Gh_t) = 0$$

$$\Rightarrow h_{xx} + R(-Ch_t - Gh) + L(-ch_{tt} - Gh_t) = 0$$

Now, the telegraph equation is rewritten as

$$h_{tt} + (\alpha + \beta)h_t + \alpha\beta h = c^2 h_{xx} \quad ,$$

where $c^2 = \frac{1}{LC}, \alpha = \frac{G}{C} \; \& \; \beta = \frac{R}{L};$

2.2 Use of Double Laplace Transform in Telegraphic Equation:

Q. Find the solution of $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2} + 2\frac{\partial z}{\partial t} + z$ (2.1)

with the conditions

$$\left.\begin{array}{l} z(x,0) = e^x; z_t(x,0) = -2e^x \\ z(0,t) = e^{-2t}; z_x(0,t) = e^{-2t} \end{array}\right\}. \quad (2.2)$$

Solution:

Invoking double Laplace transform in both sides of the PDE (2.1), we get,

$$r^2 \bar{\bar{z}}(r,s) - r\bar{z}(0,s) - \bar{z}_x(0,s)$$
$$= s^2 \bar{\bar{z}}(r,s) - s\bar{z}(r,0) - \bar{z}_t(0,s) + 2s\bar{z}(r,s) - 2\bar{z}(r,0) + \bar{z}(r,s)$$

Applying single Laplace transform on system (2.2)

$$\left.\begin{array}{l} \bar{z}(r,0) = \frac{1}{(r-1)}, \ \bar{z}_t(r,0) = -\frac{2}{(r-1)} \\ \& \ \bar{z}(0,s) = \frac{1}{(s+2)}, \ \bar{z}_x(0,s) = \frac{1}{(s+2)} \end{array}\right. .$$

Then,

$$\Rightarrow r^2 \bar{\bar{z}}(r,s) - \frac{r}{s+2} - \frac{1}{s+2} = s^2 \bar{\bar{z}}(r,s) - \frac{s}{r-1} + \frac{2}{r-1} + 2s\bar{\bar{z}}(r,s) - \frac{2}{r-1} + \bar{\bar{z}}(r,s)$$

$$\Rightarrow \bar{\bar{z}}(r,s)[r^2 - s^2 - 2s - 1] = \frac{r}{s+2} + \frac{1}{s+2} - \frac{s}{p-1}$$

$$= \frac{r^2 - s^2 - 2s - 1}{(s+2)(r-1)}$$

$$\Rightarrow \bar{\bar{z}}(r,s) = \frac{1}{(s+2)(r-1)} \quad \textbf{(2.3)}$$

Now, by taking inverse Laplace transform of (3) with respect to "s"

$$\Rightarrow \bar{z}(r,t) = \frac{e^{-2t}}{(r-1)} \quad \textbf{(2.4)}$$

Again, applying inverse Laplace transform of (4) w.r.t "r",

$$\Rightarrow z(x,t) = e^{-2t}.e^x = e^{x-2t}$$

CHAPTER 3

THE WAVE EQUATION

3.1 The wave equation [6 & 10]:

There has an impotency of a typical homogeneous type well-known hyperbolic differential equation, which is known as wave equation. Here uses the time variable "t" and one or more special variable $x_1, x_2, x_3, \ldots, x_n$ and a scalar function $g = g\ (x_1, x_2, \ldots, x_n; t)$,the model is being made by g .

Basically u is the mechanical displacement of wave. The wave equation for g is

$$\frac{\partial^2 g}{\partial t^2} = c^2 \nabla^2 g ,$$

where $\nabla = \text{nabla operator}$, $\nabla^2 = \nabla.\nabla$ is the (spatial) Laplacian operator ; and c is a constant.

This kind of differential equation appears in different branches of physics and can be found in many situations such as electro-magnetic wave, transverse vibration string, sound propagation, bar with longitudinal vibration, etc. Solution of such kind of equation is called wave function.

3.2 Use of Double Laplace Transform In Wave Equation:

Q. Find out the solution of the following wave equation,

$$\frac{\partial^2 v(x,t)}{\partial x^2} = \frac{\partial^2 v(x,t)}{\partial t^2} \qquad (3.1)$$

$$\left.\begin{aligned} &v(0,t) = 0 \\ &\frac{\partial v(x,0)}{\partial t} = 0 \\ &v(\pi,t) = 1 \\ &v(x,t_0) = 0 \end{aligned}\right\} \qquad (3.2)$$

Solution:

Taking Laplace of first two constraint of (3.2) we get

$$\left.\begin{aligned}&\bar{v}(0,q)=0\\&\&\\&\frac{\partial\bar{v}(p,0)}{\partial t}=0\end{aligned}\right\} \tag{3.3}$$

Taking Double Laplace Transformation of both sides of (3.1), we get

$$p^2\bar{\bar{v}}(p,q)-p\bar{v}(0,q)-\bar{v}_x(0,q)-q^2\bar{\bar{v}}(p,q)+q\bar{v}(p,0)+\bar{v}_t(p,0)=0 \, . \tag{3.4}$$

Using (3.3), (3.4) becomes

$$(p^2-q^2)\bar{\bar{v}}(p,q)=\bar{v}_x(0,q)-q\bar{v}(p,0)$$

$$\Rightarrow \bar{\bar{v}}(p,q)=\frac{\bar{v}_x(0,q)}{p^2-q^2}-\frac{q}{p^2-q^2}\bar{v}(p,0) \tag{3.5}$$

Let $\bar{v}_x(0,q)=\bar{F}(q) \,\&\, \bar{v}(p,0)=\bar{G}(q)$. (3.6)

By using (3.6) and Convolution theorem and Taking Inverse Laplace transform of t-variable as

$$L_t^{-1}\{\bar{\bar{v}}(p,q)\}=\bar{v}(x,q)=\frac{\bar{F}(q)}{q}\sinh(qx)-\int_0^x G(\alpha)\sinh(q(x-\alpha))d\alpha \tag{3.7}$$

Putting $x=\pi$ in (3.7) becomes

$$\frac{\bar{F}(q)}{q}=\frac{1}{q\sinh(q\pi)}+\frac{1}{\sinh(q\pi)}\int_0^\pi G(\alpha)\sinh(q(\pi-\alpha))d\alpha \tag{3.8}$$

Using (3.8), (3.7) becomes

$$\bar{v}(x,q)=\frac{\sinh(qx)}{q\sinh(q\pi)}+\frac{\sinh(qx)}{\sinh(q\pi)}\int_0^\pi G(\alpha)\sinh(q(\pi-\alpha))d\alpha-\int_0^x G(\alpha)\sinh(q(x-\alpha))d\alpha \tag{3.9}$$

Since by using usual inverse transform of (3.9) does not appear in single Laplace Transform tables, then this inversion will be effected by the evaluation of the actual integral. Accordingly,

$$v(x,t)=\frac{1}{2\pi i}\int_{B_r} e^{tq}\,\bar{v}(x,q)dq \tag{3.10}$$

where Br is Bromwich contour in the plane of integration.

Now the poles of expression $e^{tq}\bar{v}(x,q)$ are simple occurring at $q=0$ and $q=\pm ni$.
Cauchy's residue theorem can be applied

$$v(x,t)=\sum_{Residue} e^{tq}\bar{v}(x,q) \tag{3.11}$$

At $q=0$ the residue is

$$\frac{x}{\pi} \tag{3.12}$$

At $q=ni$ the residue is

$$\sum_{n=1}^{\infty} e^{nit}\frac{\sinh(nix)}{\pi(-1)^n}\left[\frac{1}{ni}-(-1)^n\int_0^x G(\alpha)\sinh(ni-\alpha)d\alpha+\int_0^\pi G(\alpha)\sinh(ni(x-\alpha))d\alpha\right] \tag{3.13}$$

At $q=-ni$ the residue is

$$\sum_{n=1}^{\infty} -e^{-nit}\frac{\sinh(nix)}{\pi(-1)^n}\left[\frac{1}{ni}-(-1)^n\int_0^x G(\alpha)\sinh(ni+\alpha)d\alpha+\int_0^\pi G(\alpha)\sinh(ni(x-\alpha))d\alpha\right] \tag{3.14}$$

Summing residue from (3.12), (3.13) & (3.14) gives

$$v(x,t)=\frac{x}{\pi}+\sum_{n=1}^{\infty}2\frac{\sinh(nix)}{\pi(-1)^n}\sin(nt)\left[\frac{1}{n}-(-1)^n i\int_0^x G(\alpha)\sinh(ni-\alpha)d\alpha+i\int_0^\pi G(\alpha)\sinh(ni(x-\alpha))d\alpha\right] \tag{3.15}$$

Since in terms of sine series $\frac{x}{\pi}$ can be written as

$$\frac{x}{\pi}=\sum_{n=1}^{\infty}\frac{2}{n\pi}(-1)^{n+1}\sin(nx) \tag{3.16}$$

Using (3.16), (3.15) becomes

$$\frac{2}{n\pi}(-1)^n\sin(nx)=\frac{\sinh(nix)}{\pi(-1)^n}\sin(nt_0)\left[\frac{1}{n}-(-1)^n i\int_0^x G(\alpha)\sinh(ni-\alpha)d\alpha+i\int_0^\pi G(\alpha)\sinh(ni(x-\alpha))d\alpha\right] \tag{3.17}$$

Putting $t=t_0$ in (3.15) and using constraint (3.2) we get

$$v(x,t_0)=0=\frac{x}{\pi}+\sum_{n=1}^{\infty}2\frac{\sinh(nix)}{\pi(-1)^n}\sin(nt_0)\left[\frac{1}{n}-(-1)^n i\int_0^x G(\alpha)\sinh(ni-\alpha)d\alpha+i\int G(\alpha)\sinh(ni(x-\alpha))d\alpha\right] \tag{3.18}$$

Using (3.16), (3.18) becomes

$$\left[\frac{1}{n}-(-1)^n i\int_0^x G(\alpha)\sinh(ni-\alpha)d\alpha+i\int_0^\pi G(\alpha)\sinh(ni(x-\alpha))d\alpha\right]=\frac{\sin(nx)}{n\sinh(nix)\sin(nt_0)} \qquad (3.19)$$

Substituting (3.19) in (3.15) gives the solution

$$v(x,t)=\frac{x}{\pi}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n\sin(nt)\sin(nx)}{n\sin(nt_0)}$$

CHAPTER 4

THE INTEGRO-DIFFERENTIAL EQUATION

4.1 Integro-differential equation [7 & 8]:

In mathematical modelling, the application of integro-differential equation plays a vital role, in various fields like as: biological occurrences, physical sciences and engineering sciences, in which it is very essential to take into account the consequence of real world problems.

Integro-differential equation gives a greater chance to get success for Varity of problems by increasing the frequency in the literature and in many other scripts on application of higher applied mathematics.

The propriety of the result and the essentiality straightforwardness of the construction of the subject, join to make the integro-differential equation reaches to a much effective value for several application.

Volterra has introduced integro-differential equation on combining integral and differential equation. It has wide applications in science and engineering. The main properties of Intergro-differential equations are

(i) Stability and (ii) boundedness of the solution.

4.2 Use of double Laplace transform in integro-differential equation:

Q. Find the solution of

$$z_x - \pi^2 z_t - z_{tt} = \pi e^{-\pi^2 t}\cos(\pi x) - t e^{-\pi^2 t}\sin(\pi x) + \int_0^t e^{-\pi^2 (t-y)} z(x,y)\,dy$$

with Conditions

$$z(x,0) = \sin(\pi x) \ \ \& \ \ z_t(x,0) = -\pi^2 e^{-\pi^2 t}\sin(\pi x);$$
$$z(0,t) = 0;$$

Solution:

Taking Double Laplace Transform in both side of the problem, we get

$$p\bar{\bar{z}}(p,s) - \bar{z}(0,s) - \pi^2 s\bar{\bar{z}}(p,s) + \pi^2\bar{z}(p,0) - s^2\bar{\bar{z}}(p,s) + s\bar{z}(p,0) + \bar{z}_t(p,0)$$
$$= \frac{1}{s+\pi^2}\bar{\bar{z}}(p,s) + \frac{\pi}{s+\pi^2}\ \frac{p}{p^2+\pi^2} - \frac{1}{(s+\pi^2)^2}\ \frac{\pi}{p^2+\pi^2}$$

$$\Rightarrow p\bar{\bar{z}}(p,s) - 0 - \pi^2 s\bar{\bar{z}}(p,s) - \frac{\pi^3}{p^2+\pi^2} - s^2\bar{\bar{z}}(p,s) + \frac{s\pi}{p^2+\pi^2} - \frac{\pi^3}{p^2+\pi^2}$$
$$= \frac{1}{s+\pi^2}\bar{\bar{z}}(p,s) + \frac{\pi}{s+\pi^2}\ \frac{p}{p^2+\pi^2} - \frac{1}{(s+\pi^2)^2}\ \frac{\pi}{p^2+\pi^2}$$

$$\Rightarrow \left[p - \pi^2 s - s^2 - \frac{1}{s+\pi^2} \right] \bar{\bar{z}}(p,s) = -\frac{s\pi}{p^2+\pi^2} + \frac{\pi p}{(s+\pi^2)(p^2+\pi^2)} - \frac{\pi}{(s+\pi^2)^2(p^2+\pi^2)}$$

$$\Rightarrow \left[p - \pi^2 s - s^2 - \frac{1}{s+\pi^2} \right] \bar{\bar{z}}(p,s) = \frac{\pi}{(s+\pi^2)(p^2+\pi^2)} \left[p - s(s+\pi^2) - \frac{1}{s+\pi^2} \right]$$

$$\Rightarrow \bar{\bar{z}}(p,s) = \frac{\pi}{(s+\pi^2)(p^2+\pi^2)}$$

Now taking Inverse Laplace Transform w.r.t "t", we get,

$$\bar{z}(p,t) = e^{-\pi^2 t} \frac{\pi}{p^2+\pi^2}$$

again applying inverse Laplace transform w.r.t "p",we obtain

$$z(x,t) = e^{-\pi^2 t} \sin(\pi x) \ .$$

Concluding Remarks:

In this dissertation we have discussed few examples and its applications by using Double Laplace Transformation. Also in fluid dynamics and elastic-dynamics that are deals with integral and partial differential equations can also be evaluate with the benefit of double Laplace transform.

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ANALYTICAL INVESTIGATION AND OPERATIONAL RULE FOR DOUBLE LAPLACE TRANSFORM

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